

THE HANDSHAKE PROBLEM **(Number Theory)**

Mathematics Standard — Number and Operations

Audience: Grades 4— Adult

Materials: None — strictly “Minds-On”

Directions: **Purpose the Question:**

If 6 people are in attendance at a meeting, and each person shakes hands with every other person only one time, what is the total number of handshakes will occur?

.....**Allow time for exploration**.....

(Encourage participants to solve the problem using any method or strategy they feel comfortable with)

Possibilities: Demonstration, drawings, narrative, discussion

Ask each group to be prepared to share their respective solutions in writing. This helps students to realize the importance of writing in mathematics as well as recording data in an organized manner.

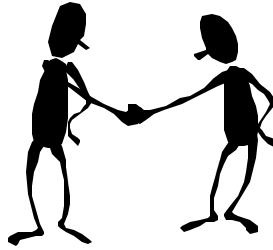
Move around the room to listen to the group discussions. You may need to guide and probe but do not give answers. If any group has difficulty getting started, ask a question or two to point them in a solid direction but no answers please!

Demonstrating solutions:

One suggestion is to issue a large poster sheet (from a flip chart) to each group for recording their result. The richness of this activity lies in the fact that in most cases, each group has a different approach to the problem. Students then begin to realize that there is more than one way to solve a problem and that ‘informal’ methods are as acceptable as formal methods. At the least, they begin to see the background and ‘why’ things work versus the isolated rote memorization of a formula.

Extensions:

Change the number of people at the meeting to 8, then 10, then 20, etc., to see if students begin to see a pattern. Students should eventually be able to make a conjecture. I have had students as early as 4 (with some guidance) learn to form a general rule.



THE HANDSHAKE PROBLEM (A Solution)

Our Problem calls for a tally of 6 people at a meeting.

Person A shakes 6 hands.

Person B shakes 5 hands since this person has already shaken hands with Person A.

Person C shakes 4 hands and so on

Do you see the pattern? Did you get 15?

1) Actually, most students will arrive at the fact that you will ultimately add (in this case):

$$5 + 4 + 3 + 2 + 1 = 15$$

For 8 people at the meeting, you will add:

$$7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$$

What you really want students to begin to see is how they can predict the solution for any number of people attending the meeting without adding all the numbers each time. This would be quite tedious for larger numbers say 20, or 50 or even 100, even with the use of a calculator!

How it works:

The final conjecture for 10 persons at the meeting is: The number of people, say 10, times the number below it, 9, = 90, divided by 2 = 45.

So it looks like $(10 \times 9) / 2 = 45$

Now try it with a new number to check your understanding. Students may want to get into a discussion of why this works. **Please note there are a variety of ways to approach this problem**



THE NEWSPAPER FOLD (A MILLION DOLLAR BET)

Audience: Grades 4 through adult

Materials: A stack of newspapers. Each participant will need the double page from any newspaper.

Areas of Mathematics Developed: Number Sense, specifically mathematical progression or exponential growth.
Division of fractions, particularly that dividing by $\frac{1}{2}$ is the same as doubling the number.

What to Do: Give each participant or group a large sheet of newspaper, a double page.
 1) Then ask everyone to fold it in half. Which way to fold it does not matter. Each individual can decide this. (1st fold)
 2) Now ask each person to fold it in half again. (2nd fold)
 3) Challenge the group to try to fold it 8 more times and offer a significant reward if anyone is successful.

I usually start the session with saying “I’ll give \$10.00 to the first person. who can fold this newspaper in half 10 times.”

Move around to make certain that each new fold is one-half of the previous fold. Have enough newspaper handy so that more than one attempt can be made if requested. Participants may try as many times as they wish until time is called.

Discussion:

Number of folds	Thickness (Layers)
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

By now you will find you are having real trouble folding the paper again. It has been getting much smaller and more importantly, much **thicker** and harder to fold. And, you have only folded it 7 times! If you can fold it again, you’ll have 256 layers of paper. You might not be able to fold it anymore, but if you can, you will have 512 layers. It is not likely you can do it the tenth time as the paper would now be 1024 layers thick!

Notes: I’ve had students to try this over and over with different sizes of paper and even with the paper wet. Encourage your audiences to explore.

Extension: See attached “25 Folds”

Answer: 33,554,432

Adapted from “Math-a-Magic” by Lawrence B. White (c. 1990) Edited and revised by Sue P. White, 6/02

**If we could fold a
sheet of paper 25
times, how thick
would it become?**

THE ALLOWANCE CHALLENGE (Number Sense)

Audience: Grades 4 — adult

Materials needed: Calculators if preferred but not totally necessary. Paper and pencil calculations are fun.

Group Arrangement: Pairs, groups of 3 each, or groups of 4 each, if the audience is large. Assign each group member some calculations to perform and then have them to cross-check each other.

This problem is an immediate attention-getter and one that eludes the participant every time! It really is designed to assess number sense and intuition for large numbers. What students will also learn in this activity is exponential growth...

Problem: WHICH WOULD YOU CHOOSE?

Choice A-----\$10,000

OR

Choice B-----A penny today, double it on Day 2,
double this on Day 3, and continue in
this manner throughout the month of
June for 30 days.

Pause and allow the group to conjecture how much Choice B would be at the end of 30 days. Record on the board (if available), on a flip chart, or on an overhead. Solicit at least 5 guesses.

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

Now, let's work the problem:

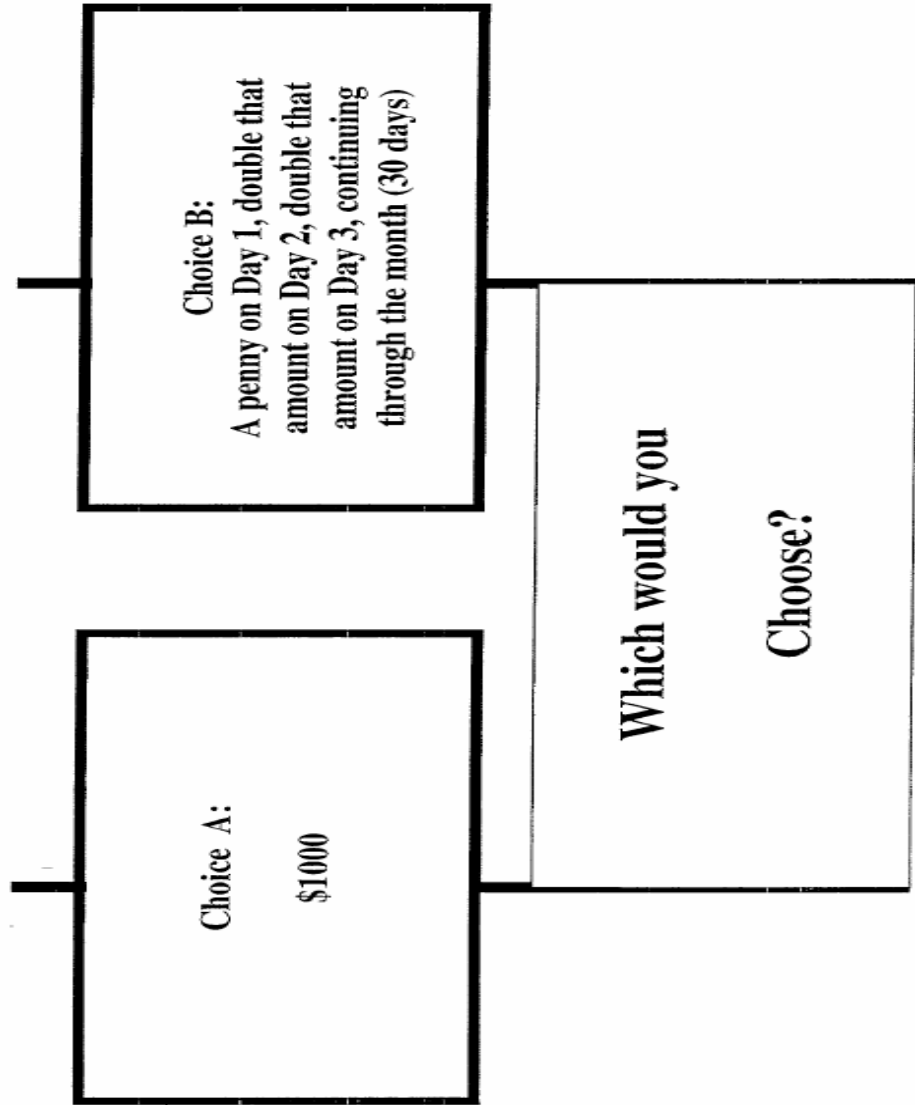
Day 1.....1 cent or \$.01
 Day 2.....2 cents or \$.02 (2 X .01)
 Day 3.....4 cents or \$.04 (2 X .02)
 Day 4.....8 cents or \$.08 (2 X .04)
 Day 5.....16 cents or \$.16 (2 X .08)
 Day 6.....32 cents or \$.32 (2 X .16)
 Day 7.....64 cents or \$.64 (2 X .32)
 Day 8.....1.28(2 X .64)
 Day 9.....\$2.56 (2 X 1.28), ETC.

Record your final answer here _____

Question: On which day do you know you've made the right or wrong decision?

- Extensions: 1) Change the dollar amount depending on the grade level. (I've used this activity successfully with students from grades 4 through middle school and in many adult workshops as well).
 2) Following the closure, ask students to think of other instances or examples where this kind of number progression will occur.

THE ALLOWANCE CHALLENGE



LESS IS BETTER

Mathematics Standard — Number and Operations

Other Ideas for Extensions

Grades Used With:

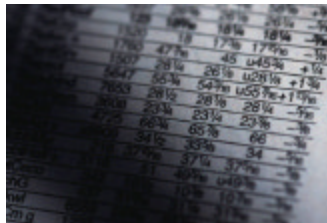
Though recommended by the original author for grade levels 5-9, with some guidance, this activity has been successfully used with younger audiences (Grades 3 and 4) and with adult audiences. In cases where children have not yet begun to study fractions, it becomes enriching and empowering for them to use their own instincts to compare the size of the fractions. They are very good at this, and you will be amazed at their wisdom!

Additional Options:

- 1) For variety, I have changed the name of the game to **MORE IS BETTER**. In this case, the obvious intent is that the person with the **highest** fraction wins. Or, worded differently, the participant is now looking for **the fraction that is closest to 1**.
- 2) My students came up with the idea of making improper fractions with the two cars to determine which is larger or smaller. This was a great idea because it stimulates even more mature audiences.
- 3) Players could also play the game without changing the fractions to equivalent decimals. Methods of comparing fractions then becomes a powerful way to check for conceptual understanding.

Editorial Comments.....

Understanding the conceptual notion of fractions still eludes even the most savvy mathematics student. This game, however, has been a fun way to get students to not only gain a deeper understanding of fractional concepts, but their relationship to decimals as well. The use of interchanging decimals and fractions in this context has more meaning and inadvertently becomes a subtle way to reinforce the isolated way it is often taught in the classrooms.



LESS IS BETTER

TIME

@ 40 MINUTES

PURPOSE

To compare fractions and decimals to see which is greater than, less than or equivalent.
Discussion of rational and irrational numbers, proper and improper fractions and patterns.

MANAGEMENT

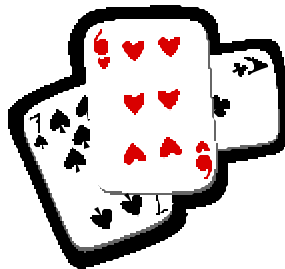
One-on-one or in pairs. Overhead cards make demonstrating the game very easy. Students love the overhead cards and love to manipulate them on the overhead so getting volunteers to help demonstrate the game is a snap. It may also help to have an overhead of the scoring sheet, because the students can duplicate it on the paper and you do not have to make copies.

AUDIENCE

Grades 5-9

MATERIALS

- Paper and pencil
- Playing cards that include Ace — 9 only, (Ace = 1)
- Score sheet (optional)
- Calculator (optional for decimal equivalents)



Reference: Kathryn Chval. All Learn Mathematics Project, University of Illinois at Chicago.
Organized and revised by Joanne Baker. 1997.

DIRECTIONS

1. Each player or team is dealt two cards.
2. The players make a proper fraction using the two cards. The player with the smallest fraction is the winner. (closest to 0)
3. Players record their fraction and equivalent decimal on the score sheet.
4. The player with the smallest fraction for the round gets one point.
5. Play continues for a specific period of time or rounds. The winner is the player with the most points. If there is a tie, players are dealt 2 more cards to see who gets the point.

VARIATIONS and EXTENSIONS

- To add to the strategy of the game, three cards are dealt to each player. For each round, the player would use two of the cards to make a proper fraction and save the third card for the next round. Strategies for using the cards wisely can be discussed.
- Players could also play the game without changing the fraction to equivalent decimals. Methods of comparing fractions could be discussed only if you want to focus in on one thing at a time.
- After a complete game is played, the players can analyze the results.
 - Which fractions were greater than $\frac{1}{2}$?
 - Which were closest to $\frac{1}{2}$?
 - Which were closest to 1?
 - Which were closest to 0?
 - Which were equivalent?
 - What is the smallest possible fraction?
 - What are all of the possible fractions using the cards 1-9?
- Players can also analyze the decimals.
 - After finding the decimal equivalence of $\frac{1}{9}$ and $\frac{2}{9}$, predict what the decimal for $\frac{3}{9}$ and $\frac{4}{9}$ will be. Check it out. Is there a pattern?
 - Investigate the other fractions and their decimal equivalence starting with $\frac{1}{x}$. Are there any of these fractions that follow a pattern?
- Non-repeating and repeating decimals can be compared.
 - What type of patterns did the students notice in the decimals?
 - Which decimals were rational and which were irrational numbers?
 - What kind of number is a fraction? (rational or irrational)
 - What makes it that?

Name/s _____

Less Is Better Scoresheet

ROUND	Player 1	Player 2	Player 3	Player 4	WINNER
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

COUNTER GAME

Mathematics Standard — Data Analysis and Probability

Ideas for Extensions

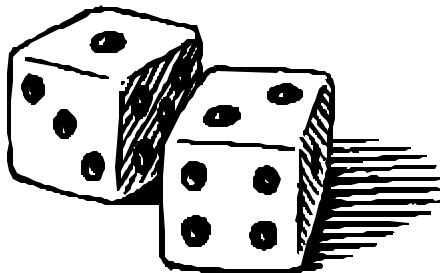
Audience: Although the author recommended this game for grades 6 — 8, trainers have used it with successfully with grades 4 through 8. Students love this game and clearly understand the essence of the probabilities. Discussions that follow usually lead into the fairness of such games of chance. Students have often brought in their own games from home and have further explored probabilities of winning matched with maximizing strategies for winning.

Other Variations:

- 1) When rolling the dice, use the ‘difference’ between the cubes. If you choose to do this, be sure to add a column for zero to allow for rolling doubles. This is an interesting ‘twist’ as students will realize the limitations of this choice.
- 2) Use multiplication and record the ‘products’ of the two numbers rolled. Students will then have to list the possibilities in order to generate a new column format. What are the most frequently occurring products?
- 3) Use Fraction Dice and this gives the game a new flavor! Try all the operations. This is a great variation for reinforcing “Operations with Fractions”.

Editorial Comments.....

The National Mathematics Standards published by the NCTM reflect that students don't get nearly enough 'hands-on' experiences relative to learning probability. Trainers should use games as a fun and interesting resource to support informal learning in this area.



COUNTER GAME

TIME

@ 40 MINUTES

PURPOSE

Think about strategies for winning and the probabilities associated with those strategies.

MANAGEMENT

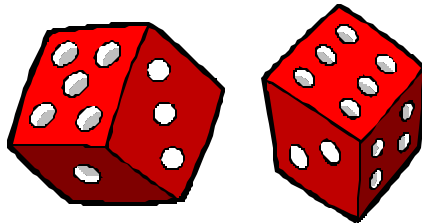
Two team game, can have one on one, or pairs on a team so the students have someone to discuss their strategies with.

AUDIENCE

Grades 6-8

MATERIALS

- Counter Game Board, (or a paper that has columns numbered 1 through 12)
- Pair of number cubes (numbered 1-6) or a pair of dot cubes 1-6 per team
- 12 markers per team so each team has different colors or sizes to distinguish the team markers like buttons, counters, cm cubes, pennies (heads, tails)



Reference: Adapted from Lappan, Fey, Fitzgerald, Friel, and Phillips. Connected Mathematics. Data. How Likely Is It? Dale Seymour Publications. Palo Alto, CA. 1996. Joanne Baker. 1997.

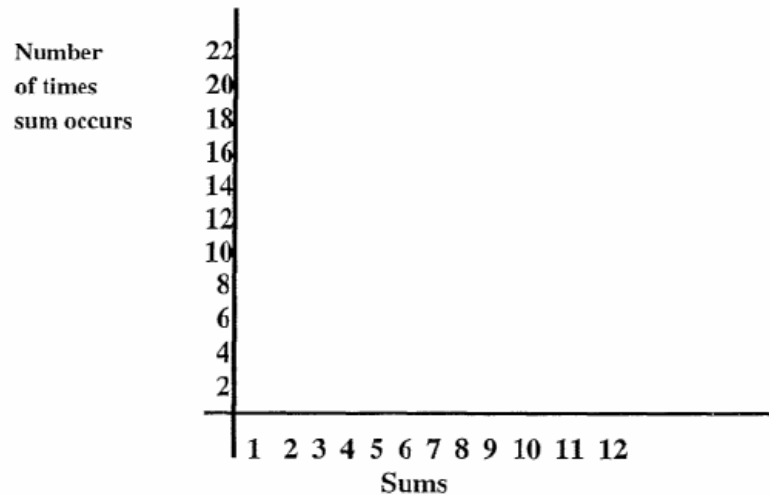
DIRECTIONS

1. Each team places it 12 markers into the columns in any way they choose.
2. Each team rolls a number cube. The team with the highest roll goes first.
3. Teams take turns rolling the two numbers cubes and moving a marker from the column with the same number as the total shown on the cubes. (the sum) If the column is empty, the team does not get to remove a marker.
4. The first team to remove all the markers from its board wins.

VARIATIONS and EXTENSIONS

Find a systematic way to list all the possible outcomes(number pairs) of rolling two number cubes and the sums of each of these outcomes. Analyze your list carefully.

If you are having trouble organizing your data set up a graph to assist you in answering the following questions.



Questions to Ask

- What sums are possible when you roll two cubes?
- Which sum or sums occur most often?
- How many ways can you get a sum of 6? 8?
- A sum of 2?
- Are all the sums equally likely? Explain.

Play again using any new strategies for winning. Did it help?

